CALCULATING SURFACE AREA TO VOLUME RATIOS

- First of all, the **SURFACE AREA** (abbreviation = **SA**) is the area of material that it would take to completely cover an object (for example, the amount of wrapping paper it would take to precisely wrap an expensive gift to your biology teacher 😊).
- For a **simple cube shape**, surface area is simply the length of one side times the height of one side times the number of sides (all cubes have, incidentally, 6 sides). The formula can be written like this.

\[
\text{Surface Area} = \text{Length} \times \text{Height} \times 6
\]

- For example, a cube that was 3 cm on each side would have a surface area of:

\[
\text{SA} = 3 \text{ cm} \times 3 \text{ cm} \times 6 = 54 \text{ cm}^2
\]

Notice that the units are "square" units.

- **VOLUME** (V) is the amount of space inside an object.
- Volume for a simple cube is equal to the length times the height times the depth of an object.

\[
V = L \times H \times D
\]

For the above example of a 3 cm cube, the volume would be:

\[
V = 3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm} = 27 \text{ cm}^3
\]

Notice that the units are "CUBIC" units.

The **surface area to volume ratio** (SA:V) is the proportion of area of material covering an object to amount of material within the object. i.e. it is a measure of how much covers the object for each unit of interior volume.

For the above example:

\[
\frac{\text{SA}}{V} = \frac{54}{27} : \frac{27}{27}
\]

2:1

- The surface area to volume ratio for the above cube is equal to 2:1. That is, for every 2 units of surface area covering the outside of the cube, there is 1 unit of volume in the inside of the cube. If this cube were a cell, we could further say that there are 2 units of cell membrane for every unit volume of cytoplasm in its interior.

**Now you try:** Key is on other side -- **DO NOT LOOK** until you've solved these problems on your own.

- For the four cubes below, **measure** (in cm) each with a ruler, and **calculate** the surface area, the volume, and the surface area to volume ratio. Reduce the SA:V ratio so that it the "V" is = 1.

<table>
<thead>
<tr>
<th>Cube A</th>
<th>Surface Area</th>
<th>Volume</th>
<th>S:V ratio (reduced)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube B</td>
<td>Surface Area</td>
<td>Volume</td>
<td>S:V ratio (reduced)</td>
</tr>
<tr>
<td>Cube C</td>
<td>Surface Area</td>
<td>Volume</td>
<td>S:V ratio (reduced)</td>
</tr>
<tr>
<td>Cube D</td>
<td>Surface Area</td>
<td>Volume</td>
<td>S:V ratio (reduced)</td>
</tr>
</tbody>
</table>

1. What trend do you notice as the size of the cells increases?
2. Why is this important to living cells?
3. What can cells do about it?
KEY: You're not sneaking a peek before attempting the questions, are you?

<table>
<thead>
<tr>
<th>Cube A</th>
<th>Surface Area</th>
<th>1.5 cm²</th>
<th>Volume</th>
<th>0.125 cm³</th>
<th>S:V ratio (reduced)</th>
<th>12:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube B</td>
<td>Surface Area</td>
<td>6 cm²</td>
<td>Volume</td>
<td>1 cm³</td>
<td>S:V ratio (reduced)</td>
<td>6:1</td>
</tr>
<tr>
<td>Cube C</td>
<td>Surface Area</td>
<td>24 cm²</td>
<td>Volume</td>
<td>8 cm³</td>
<td>S:V ratio (reduced)</td>
<td>3:1</td>
</tr>
<tr>
<td>Cube D</td>
<td>Surface Area</td>
<td>96 cm²</td>
<td>Volume</td>
<td>64 cm³</td>
<td>S:V ratio (reduced)</td>
<td>1.5:1</td>
</tr>
</tbody>
</table>

1. What trend do you notice as the size of the cells increases? The surface area to volume ratio decreases as cell size increases (e.g. as cell size doubles, SA:V ratio is reduced 50%)

2. Why is this important to living cells? Living cells can only get materials in (like glucose and oxygen) through the cell membrane, and can only get waste products out through the cell membrane. As SA:V ratio decreases, it means that every unit of volume of cytoplasm is "serviced" by less and less surface area. If the cell is actively metabolizing, it may not be able to get in enough nutrients or get rid of its wastes. Furthermore, as most things get around in cell via diffusion (a free albeit slow process), as a cell gets larger, it may not be able to get materials from one end of a cell to the next by diffusion quick enough to sustain its chemical reactions.

3. What can cells do about it? Cells can:
   - divide
   - slow down metabolism
   - change shape to increase the surface area (e.g. get long and thin or flat, or develop many folds in the cell membrane to increase SA and therefore SA:V ratio.)